Lab no. 8

SINGLE-PHASE BRIDGE RECTIFIERS WITH CURRENT FILTERS

1. Introduction

The single-phase full-bridge structure (B2) from the natural commutation (line commutated, line-frequency) rectifiers class is used in the low-power applications, preferably up to 1kW. Compared with the three-phase bridge, the single-phase bridge is simpler, cheaper and more easily controllable. On the other hand, because the single-phase rectifiers perform only a two-pulse rectification, the pulses' frequency from the output voltage waveform is low and large current or voltage filters are required.

A two-pulse rectification can also be achieved using a single-phase midpoint rectifier (M2) which requires obligatorily an oversized line-frequency transformer with two identical secondary windings. Unlike the M2 midpoint structure, the B2 bridge structure can be supplied directly from the power utility grid if a voltage level adaptation is not necessary. In this way, it is avoided an expensive circuit element with high size and mass as the low-frequency transformer. If an adaptation of the voltage level and/or an electrical isolation is required, in a modern variant, it is preferred the direct rectification of the grid voltage after that a DC/DC converter, with or without electrical isolation is used. Thus, we obtain the modern switch-mode power supplies (SMPS) with or without a high-frequency transformer.

An additional argument in favour of the bridge rectifiers is the possibility of finding easily and at a low price power integrated modules (PIM) that integrate this structure for a wide range of voltages and currents.

2. Topology of the single-phase bridge rectifier

As shown in Fig.8.1, the single-phase full-bridge rectifier consists of two legs, each leg consisting of two serially connected semiconductor devices, rectifier diodes for the ***uncontrolled rectifiers*** or thyristor for the ***phase-controlled rectifiers***. Thus, result four rectifier devices, two more than the M2 rectifier. This difference is acceptable if it takes into consideration the advantages of the bridge structure mentioned above and the advantage of using the rectifier devices with a smaller maximum reverse voltage.

In Fig.8.1 it is presented the most general case of a phase-controlled B2 rectifier (with thyristors) supplied through a line-frequency transformer. The DC load

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that requires a current filter can be of a resistive-inductive *RL* type (Fig.8.1.a) or of *RLE* type (Fig.8.1.b). The latter is the active DC load which include their own power source. The most representative example of this category are the DC motors whose equivalent circuit contains, besides the armature resistance R_a and inductance L_a, the *emf* voltage *E*. If the load inductance is not high enough for a proper current filtering, it is used an additional ***filter inductor***, labelled L_f.

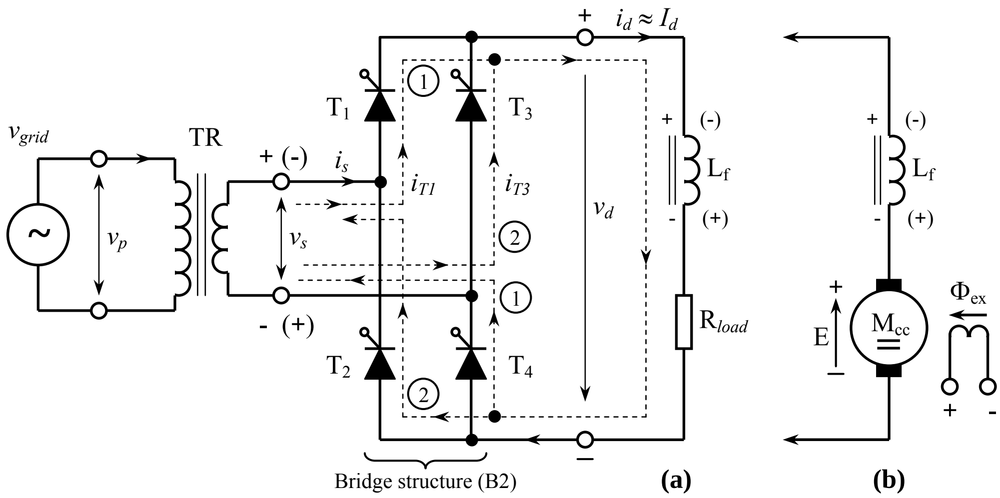

Fig. 8.1 Single-phase full-bridge rectifier with: (a) R-L load; (b) R-L-E load.

3. Analysis of the uncontrolled single-phase rectifier (with diodes)

If the power structure from Fig.8.1 is performed with diodes instead of thyristors, we obtain an uncontrolled bridge rectifier with an output current filter. Its operation can also be obtained using a thyristor rectifier if the firing angle is $\alpha=0^o$. Thus, the analysis of the uncontrolled rectifier with current filter becomes a particular case of the phase-controlled rectifier operation. Fig.8.2 shows the waveforms corresponding to this case.

In the first diagram of the supply voltage u_s (transformer secondary voltage) are figured the following natural commutated points:

- **P** for the thyristors forward biased on the positive half wave (cycle) $\rightarrow T_1, T_4$;
- **N** for the thyristors forward biased on the negative half wave (cycle) (polarity in the brackets on the transformer's secondary terminals) $\rightarrow T_3, T_2$;

In the case of a continuous current conduction mode and a delay (firing) angle $\alpha=0^o$, the waveform of the rectifier output voltage is made of positive half sine waves due to the rectification process of the bridge. The instantaneous values of v_d voltage is given by the following equation:

$$v_d(\omega t) = |v_s(\omega t)| = \begin{cases} v_s(\omega t) \text{ for } 0 \le \omega t < \pi \quad (\mathrm{T_1, T_4 \text{ on}}) \\ -v_s(\omega t) \text{ for } \pi \le \omega t < 2\pi \ (\mathrm{T_2, T_3 \text{ on}}) \end{cases} \tag{8.1}$$

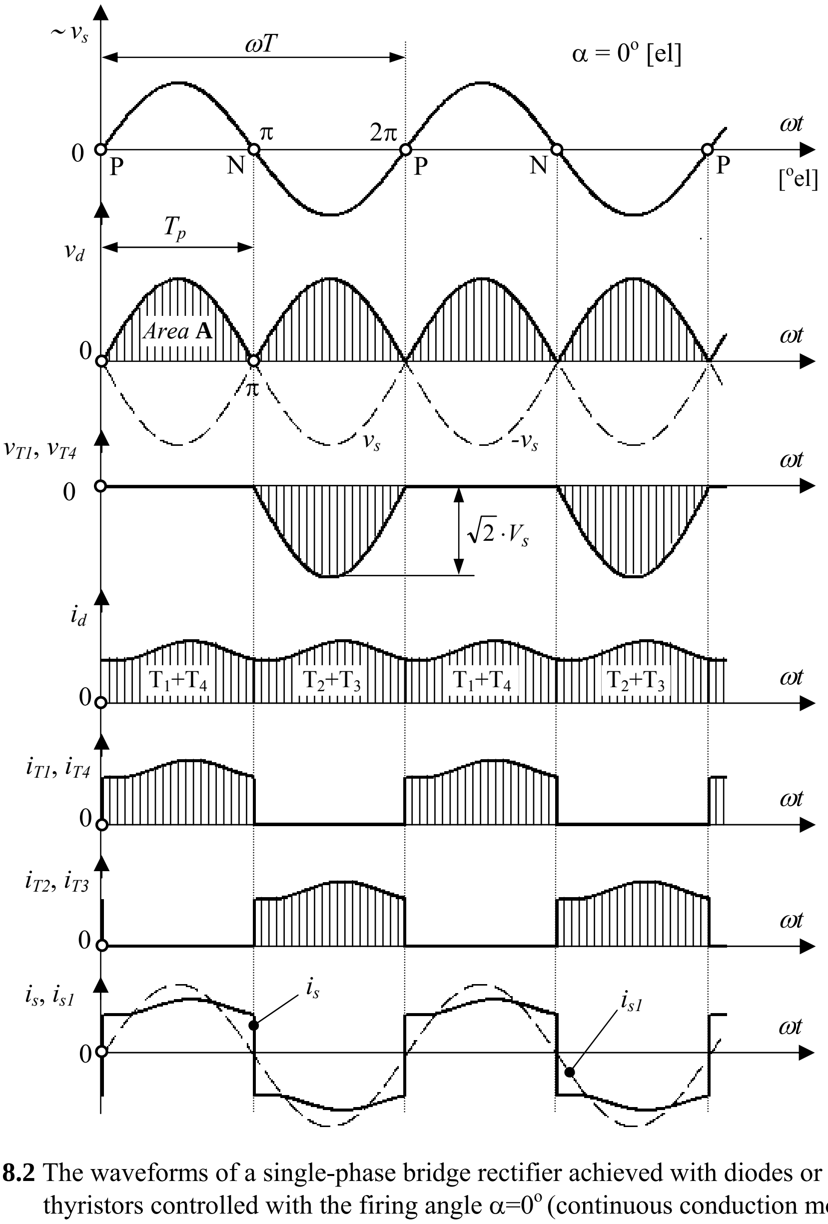

Fig. 8.2 The waveforms of a single-phase bridge rectifier achieved with diodes or with thyristors controlled with the firing angle α=0° (continuous conduction mode).

In the equation (8.1) and in Fig.8.2 it was considered an ideal current commutation from the path of the thyristors (T_1, T_4) to the path of thyristors (T_2, T_3) and vice versa (duration of the natural commutation process is zero).

To calculate the DC component of the output voltage v_d (DC voltage) it is applied the average formula during the T_p time period of a voltage pulse (π radians):

$$\begin{aligned} V_{d0} &\overset{not}{=} \text{average value of } v_d(t) = \frac{1}{T_p}\int_0^{T_p} v_d(t)\cdot dt = \frac{1}{T_p}\cdot Area\ \mathrm{A} = \\ &= \frac{1}{\pi}\int_0^{\pi} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\int_0^{\pi}\sqrt{2}V_s\sin(\omega t)\cdot d(\omega t) = \\ &= \frac{\sqrt{2}V_s}{\pi}\left[-\cos(\omega t)\right]_0^{\pi} = \frac{\sqrt{2}V_s}{\pi}\left[\cos 0 - \cos\pi\right] = \\ &= \frac{2\sqrt{2}V_s}{\pi} \cong 0{,}9\cdot V_s \end{aligned} \tag{8.2}$$

Equation (8.2) highlights the relationship between the DC voltage V_{d0} at the output of a rectifier with diodes (or with thyristors controlled $\alpha = 0^o$) and the effective value of the input AC voltage V_s. It is noted that, the maximum value of the DC voltage provided by a single-phase rectifier cannot reach the effective voltage of the input AC voltage ($V_{d(\max)} = V_{d0} \approx 0{,}9\cdot V_s$).

The filter inductance L_f, no matter how great is, cannot perfectly smooth the current $i_d(t)$ as shown in Fig.8.2, where the waveform of the output current contains an AC component due to voltage pulses of the $v_d(t)$. For the rectifier analysis we suppose that the output current is well filtered and we can write: $i_d(t) \approx I_d = constant$, where the I_d is the average value. The bridge rectifier topology from Fig.8.1 and the current waveforms from Fig.8.2 show that during a half-cycle time interval $T_p = T/2$ the I_d current flows through the T_1 and T_4 thyristors path, denoted by (1) and in the next half-cycle the current flows through the T_2 and T_3 thyristors path, denoted by (2). During the conduction of the pair (T_1, T_4), the current i_s in the AC side is positive and during the conduction of the pair (T_2, T_3), it becomes negative.

If we suppose that $i_d(t) \approx I_d = constant$ we can calculate the rms value of the $i_s(t)$ input current based on the average value of the output current I_d:

$$I_s = \sqrt{\frac{1}{T}\int_0^T i_s^2(t)\cdot dt} = \sqrt{\frac{1}{T}\left(\int_0^{T/2} I_d^2\cdot dt + \int_{T/2}^{T}\left(-I_d\right)^2\cdot dt\right)} = I_d \tag{8.3}$$

The non sinusoidal waveform of the current i_s from the AC side suggests that the line commutated rectifiers pollute the AC power grid with current harmonics because besides the fundamental component of the input current i_{s1} many other current harmonics appear.

4. Analysis of the controlled single-phase rectifier (with thyristors)

Fig.8.3 shows the waveforms corresponding to a single-phase bridge converter with thyristors that operates in rectifier and in continuous conduction mode ($\alpha = 60^o$).

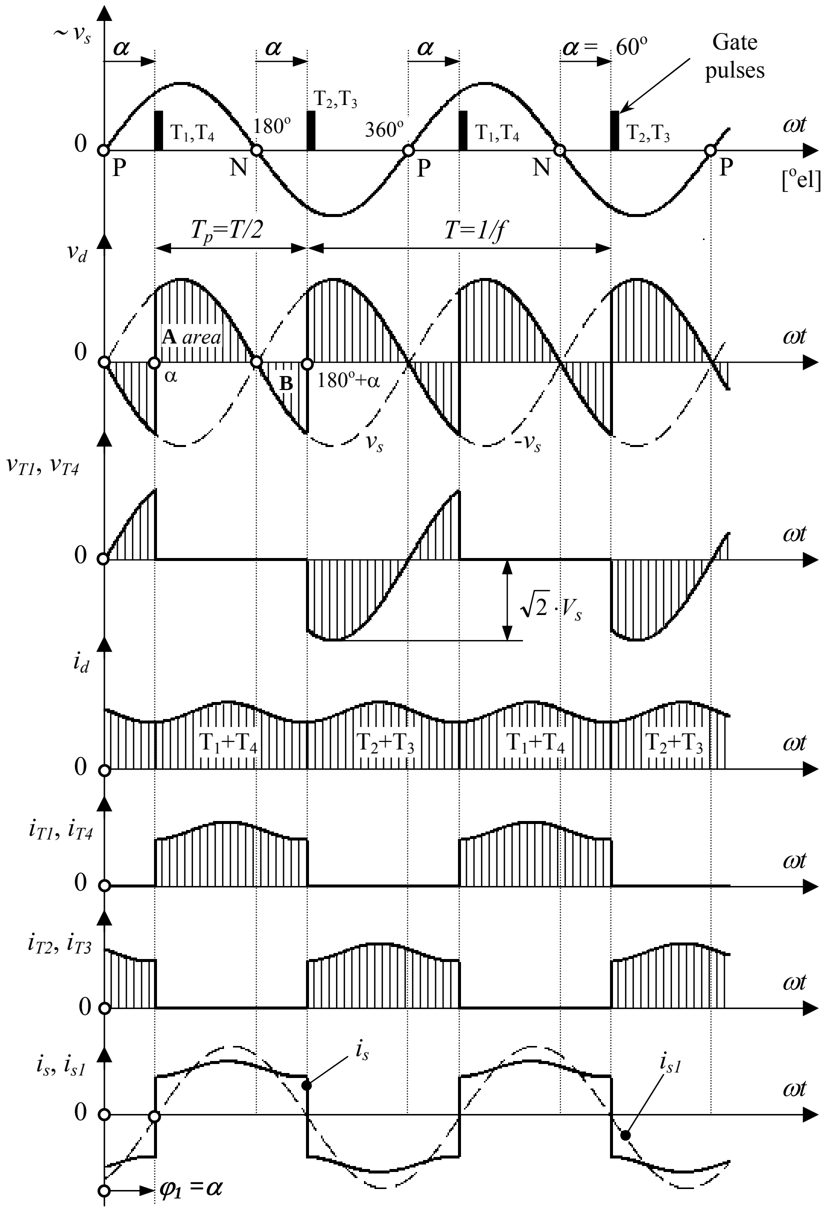

Fig. 8.3 Waveforms corresponding to a single-phase bridge rectifier with thyristors controlled with the firing angle α=60° [el].

In the case of a continuous conduction mode of the rectifier and in the case of an ideal current commutation, the output v_d voltage waveform is made of pulses whose instantaneous values are given by:

$$v_d(\omega t) = \begin{cases} v_s(\omega t) \ \text{ for } \ \alpha \le \omega t < \pi + \alpha & (\mathrm{T}_1, \mathrm{T}_4 \text{ on}) \\ -v_s(\omega t) \ \text{ for } \ \pi + \alpha \le \omega t < 2\pi + \alpha & (\mathrm{T}_2, \mathrm{T}_3 \text{ on}) \end{cases} \tag{8.4}$$

During the time period $T = 1/f$ of the input AC voltage, the rectifiers provide two identically voltage pulses with the time period $T_p = T/2$. To calculate the DC component of the output voltage, it is applied the average formula on the T_p period (π radians):

$$V_{d\alpha} \overset{not}{=} \text{average value of } v_d(t) = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\cdot\left(Area\ \mathrm{A} + Area\ \mathrm{B}\right) =$$

$$= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\sqrt{2}V_s \sin(\omega t)\cdot d(\omega t) = \frac{\sqrt{2}V_s}{\pi}\left[-\cos(\omega t)\right]_{\alpha}^{\pi+\alpha} = \tag{8.5}$$

$$= \frac{\sqrt{2}V_s}{\pi}\left[\cos\alpha - \cos(\pi+\alpha)\right] = \frac{2\sqrt{2}V_s}{\pi}\cdot\cos\alpha = V_{d0}\cdot\cos\alpha$$

Equation (8.5) highlights that a phase-controlled rectifier operating in continuous conduction mode provides at the output an average voltage (DC voltage) regulated exclusively through the delay (firing) angle α (controlled rectifier).

If we look the rectifier as a system we can define the transfer function or the ***adjustment (control) characteristic*** $V_{d\alpha} = f(\alpha)$ as shown in Fig.8.4. This is a non-linear characteristic due to the cosine function from the DC voltage expression. The conclusion is valid for all phase-controlled rectifiers.

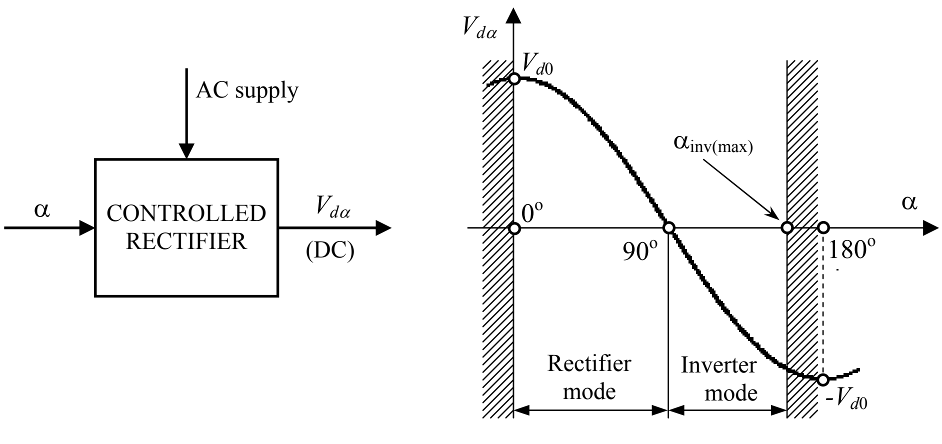

Fig. 8.4 Adjustment characteristic of a phase-controlled rectifier: $V_{d\alpha} = f(\alpha) = V_{d0}\cdot\cos\alpha$.

As shown in Fig.8.4, the nonlinearity of the transfer function is substantially reduced if the firing angle is adjusted in a range around 90°el. Since the firing angle is limited to a maximum value $\alpha_{inv(max)}$ in inverter mode (as a consequence of the natural commutation process of the current), in order to maintain the symmetry of the maximum amplitude of the positive and negative DC voltage, the firing angle is limited also to a minimum value $\alpha_{rect(min)}=180^o$ - $\alpha_{inv(max)}$ in rectifier mode. Thus, when we use rectifiers in controlled systems (e.g. electric drives with DC motors), in order to avoid a non-linear transfer characteristic of the whole system, it is preferred to adjust the firing angle in the range of:

$$\alpha_{redr(min)} \leq \alpha \leq \alpha_{inv(max)}. \quad (8.6)$$

Usually, the range of firing angle is: $30^o \leq \alpha \leq 150^o$

Analyzing the v_d voltage waveform of Fig.8.3, it is observed that once the firing angle increases above 0^o, the voltage pulses contain portions of negative half waves. Therefore:

- Average voltage gradually decreases, as shown in Fig.8.4, reaching the zero value when the firing angle is $\alpha = 90^o$. For $0^o \leq \alpha < 90^o$ the converter operates in ***rectifier mode***. In the range of $90^o < \alpha \leq \alpha_{inv(max)}$ the converter can operate in ***inverter mode*** ($V_{d\alpha} < 0$) if other special conditions are met. This mode of operation of the phase-controlled rectifiers will be treated in the next laboratory application (Lab no.9).
- If the inductance from the DC side of the rectifier is missing or is insufficient, the discontinuous current conduction mode may occur, the operating mode treated in the next sections.

For the waveforms shown in Fig.8.3 the L_f filter inductance was considered large enough so that the rectifier discontinuous conduction mode is avoided and the load current is relatively well filtered. Despite the ripple of the i_d current for the rectifier analysis, we use the average value I_d. This current is commutated cyclically between the path denoted by (1) and the path denoted by (2) in Fig.8.1. Always are on (conduction state) a top thyristor from a bridge leg with a bottom thyristor from the other leg. During the conduction of (T_1,T_4) pair the input i_s current is positive and during the conduction of (T_2,T_3) pair this current is negative.

Analyzing the i_s waveform we can observe that the phase-controlled rectifiers pollute the utility power grid in several ways. Besides the harmonic pollution (grid voltage distortion) due to non-sinusoidal current waveform and the line notching induced in the grid voltage waveform due to the short circuit of the natural commutation process (not shown in Fig.8.3), it is absorbed a reactive power named ***control reactive power***:

$$Q = V_s \cdot I_{s1} \cdot \sin\varphi_1 = V_s \cdot I_{s1} \cdot \sin\alpha \quad (8.7)$$

Due to phase displacement $\varphi_1 = \alpha$ between the fundamental harmonic $i_{s1}(t)$ of the AC current and AC supply voltage (see Fig.8.3).

5. Discontinuous conduction mode of operation for the single-phase controlled rectifier

All the analysis that have been made in the previous sections regarding the single-phase rectifier with current filters, have started from the assumption of a normal operation in which the output DC current is well filtered and it is continuous through the converter (continuous conduction mode). Not always this type of operation mode can be obtained. The permanent existence of the current in the DC side and implicitly during the entire conduction time interval allocated for the diodes or thyristors is based on the energy accumulated in the whole inductance connected to the rectifier's output (load inductance). If this inductance is insufficient or the load current I_d falls below a minimum value $I_{d(min)}$, the energy stored in the inductance ($W_L = L \cdot I_d^2 / 2$) decreases under a certain value that can't assure the devices' conduction during the entire time interval in which the input AC voltage or the active load voltage tends to reverse bias these devices. In this situation, the current is interrupted through the rectifier and the ***discontinuous conduction mode*** appears. In this mode, the waveforms presented in the previous section will change depending on the rectifier load type. Also, some negative consequences, which will be mentioned later, appear.

a) Discontinuous conduction mode in case of a rezistive load

A clear and suggestive discontinuous conduction mode appears when a purely resistive load is used by the phase-controlled rectifier, as presented in Fig.8.5.

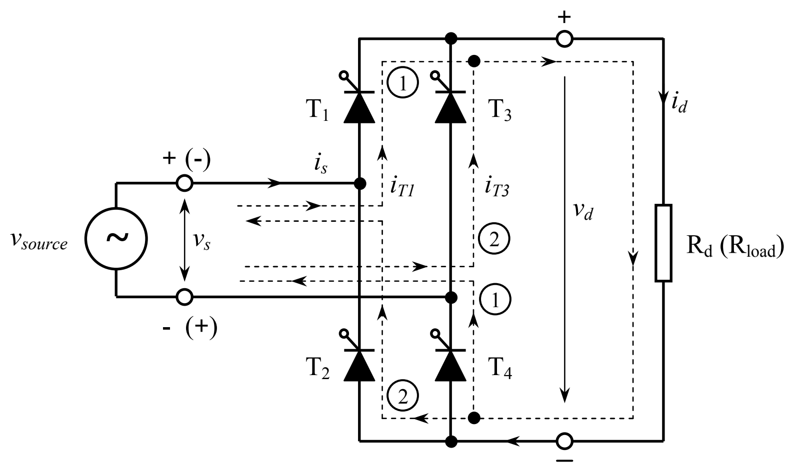

Fig. 8.5 Phase-controlled rectifier with a purely resistive load.

This application is rarely used in practice because the DC loads are usually of type *R-L* and *R-L-E*. However, the analysis of a phase-controlled rectifier with resistive load is imposed by theoretical and didactic reasons because it is the simplest application from a lot of applications with controlled rectifiers. If all the circuit inductances are neglected (of the AC source, connection conductors and also of the load circuit), the i_d output current follows exactly the waveform of the u_d output voltage, as shown in Fig.8.6.

If the thyristors of the rectifier are controlled with the firing angle $\alpha = 0^o$ or the rectifier is uncontrolled, with diodes, the i_d current is continuous (not interrupted). Contrary, if the firing angle is not zero ($\alpha > 0^o$), the conduction of every pair of thyristors (T_1,T_4) and (T_2,T_3) occurs until the AC input voltage cross the zero value, because there is not an inductance that can maintain the current flow on the negative voltage based on the energy accumulated in its field. Therefore, the i_d current becomes discontinuous (interrupted) and the devices in conduction are turned off. Consequently, during these off time intervals, the rectifier DC side is disconnected from the AC side until the next pair of thyristors is turned-on. During the time interval in which the i_d current is zero, the instantaneous v_d voltage becomes also zero:

$$v_d(\omega t) = 0 \quad \text{for} \quad k\pi < \omega t \le k\pi + \alpha, \quad k = 0, 1, 2, 3 ..... \tag{8.8}$$

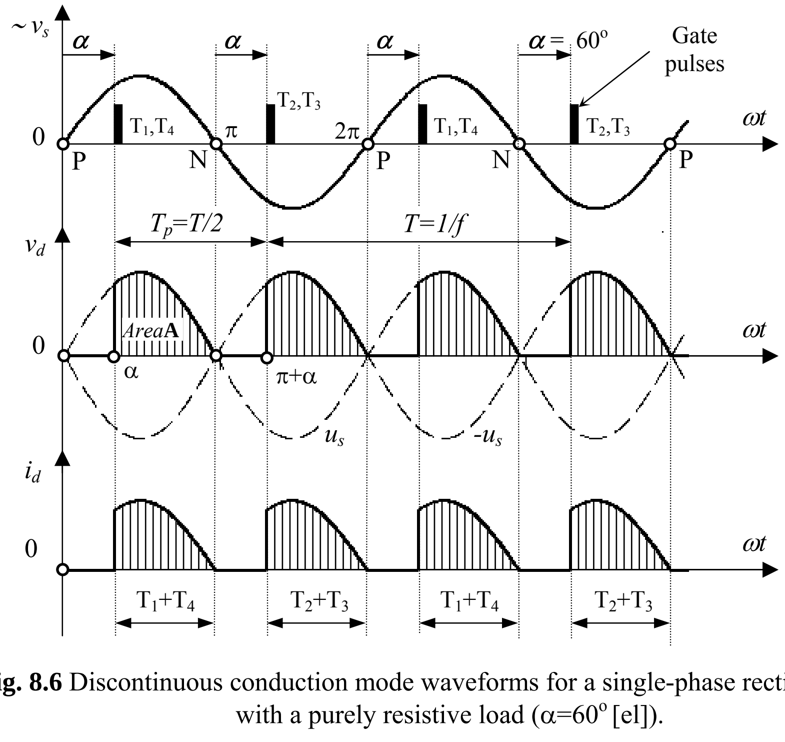

Fig. 8.6 Discontinuous conduction mode waveforms for a single-phase rectifier with a purely resistive load (α=60° [el]).

In such situations, the time intervals where the rectifier operates in instantaneous inverter mode with negative voltage pulses (u_d <0), disappear (see Fig.8.3). It's obvious that the voltage's average value at the rectifier's output will change:

$$\begin{aligned} V_{d\alpha}^{'} &= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi} v_d(\omega t)\cdot d(\omega t) + \frac{1}{\pi}\int_{\pi}^{\pi+\alpha} 0\cdot d(\omega t) = \\ &= \frac{1}{\pi}\cdot Area\ \mathrm{A} = \frac{1}{\pi}\int_{\alpha}^{\pi}\sqrt{2}V_s \sin(\omega t)\cdot d(\omega t) = \frac{\sqrt{2}V_s}{\pi}\left[-\cos(\omega t)\right]_{\alpha}^{\pi} = \\ &= \frac{\sqrt{2}V_s}{\pi}\left(\cos\alpha - \cos\pi\right) = \frac{\sqrt{2}V_s}{\pi}\cdot\left(1+\cos\alpha\right) = \frac{V_{d0}}{2}\cdot\left(1+\cos\alpha\right) \geq 0 \end{aligned} \tag{8.9}$$

Note that, in the discontinuous conduction mode of a phase-controlled rectifier with resistive load, the average output voltage is always positive and higher than the DC voltage provided by the same rectifier that operates in continuous conduction mode with the same α firing angle:

$$V_{d\alpha}^{'} \geq V_{d\alpha} \Leftrightarrow \frac{V_{d0}}{2}\left(1+\cos\alpha\right) \geq V_{d0}\cdot\cos\alpha \Leftrightarrow 1 \geq \cos\alpha \ \rightarrow \ \text{true} \tag{8.10}$$

The relation (8.10) is confirmed also by the Fig.8.7 where are comparatively presented the adjusting characteristic ($V_{d\alpha}=f(\alpha)$) of a phase-controlled rectifier with filter inductance operating in continuous conduction mode and the adjusting characteristic ($V_{d\alpha}'=f(\alpha)$) of a phase-controlled rectifier with purely resistive load (or with recovery diode at the output) operating in discontinuous conduction mode.

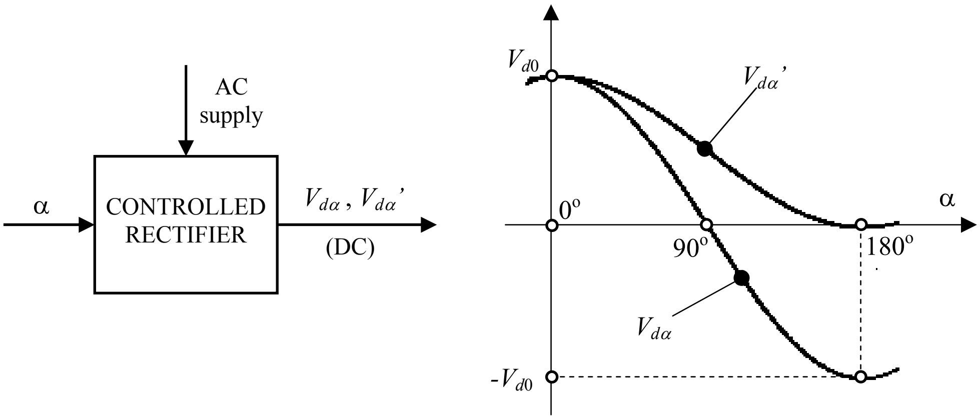

Fig. 8.7 $V_{d\alpha}=f(\alpha)$ for a rectifier with filter inductance and continuous conduction mode;
$V_{d\alpha}' = f(\alpha)$ for a rectifier with resistive load and discontinuous conduction mode.

There is not a natural current commutation at the phase-controlled rectifiers which operate in discontinuous conduction mode because the i_d current is zero before the thyristors turn-on. Consequently, there is no problem in limiting the maximum firing angle like in a continuous conduction mode. So, a rectifier with a resistive load or with a recovery (antiparallel) diode at the output can be controlled in the whole range of the firing angle ($0^o \div 180^o$), always obtaining a positive average voltage with values that follow the adjusting characteristic $V_{d\alpha}' = f(\alpha)$ shown in Fig.8.7. In this case the phase-controlled rectifier cannot operate in the inverter mode.

b) Discontinuous conduction mode in case of a resistive-inductive load

There are passive DC loads which need a filtered DC current if they are supplied with pulses voltages with a DC component as the rectifiers provides. In this case, besides the inductance proper to the load (L_{load}), which many times is insufficient, a filtering inductance L_f is added. So, from the rectifier's point of view, the load is of a resistive-inductive type, as shown in Fig.8.8 where:

$$L_d = L_f + L_{load} \quad \text{and} \quad R_d = R_f + R_{load}$$

The ***filter inductance*** is sized according to the ***maximum ripple amplitude*** of the i_d current and according to the ***minimum current value*** $I_{d(min)}$ that can occur in the load circuit. Above this value the rectifier must operate in continuous conduction mode. It is desired that $I_{d(min)}$ value to be as high as possible to reduce the filter inductance and thus to reduce the size, mass and the price of the rectifier.

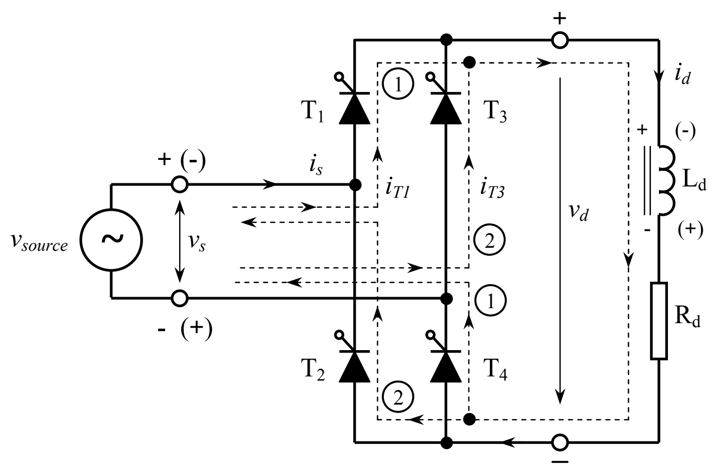

Fig. 8.8 Phase-controlled rectifier with resistive-inductive load.

Observe that the filter inductance value L_f is the result of a compromise and consequently there is a load current threshold value I_d^* under which the discontinuous conduction mode occurs. The waveforms of this mode for an *R-L* load type are shown in Fig.8.9.

The first part of the conduction (on) time intervals corresponding to the thyristors pairs (T_1,T_4) and (T_2,T_3), the current $i_d(t)$ has an increasing evolution ($di_d / dt > 0$) and the load inductance L_d stores energy in its electromagnetic field. In the second time subinterval where the current slop is negative ($di_d / dt < 0$) and the value of the supply voltage decreases, the inductance starts to provide energy from its field. After zero voltage crossing, there is a time interval where the rectifier operates in instantaneous inverter mode, when the current flow extends also during the opposite half wave. If the stored energy in the inductance field is relatively low, these time intervals aren't prolonged until the other pair of thyristors is turned on. So, the discontinuous conduction mode appears.

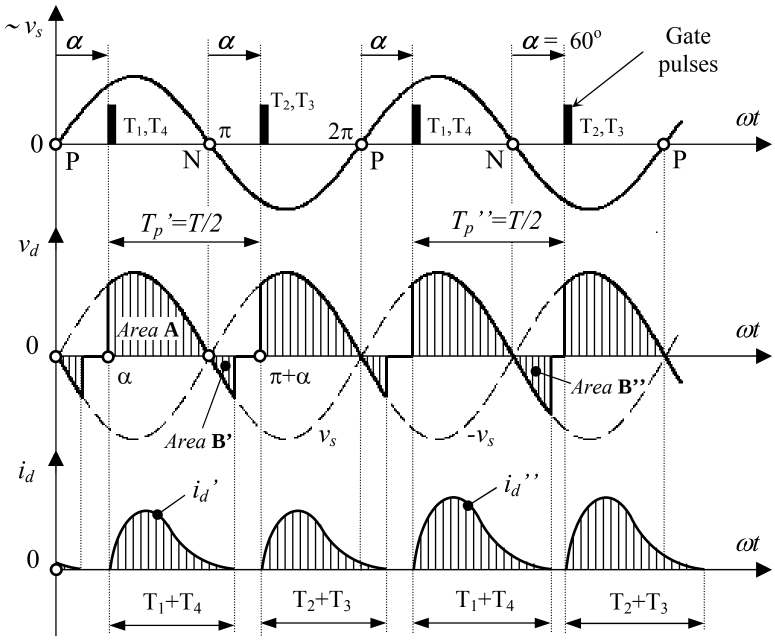

Fig.8.9 Discontinuous conduction mode in the case of a single-phase rectifier with a resistive-inductive load (α=60° [el]).

The discontinuous current conduction is an operation mode not recommended because of the following issues:

- Abnormal function of the load sensitive to the DC current waveform. If the load requires a current filter, it must therefore require a smooth current waveform and a discontinuous current can affect its proper functioning.
- The rectifier becomes uncontrollable because the output DC voltage depends, besides the α delay angle, on the load current I_d. To highlight this effect, we

write the equations of the average output voltage for two different currents $i_d^{'}(t)$ and $i_d^{''}(t)$ which have the waveforms shown in Fig.8.9:

- *In the case of* $i_d^{'}(t)$ *current* the direct voltage is calculated with the average value formula for the $T_p^{'}$ time interval (pulse time period):

$$V_{d\alpha}^{'} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\cdot\left(Area\ \mathrm{A} + Area\ \mathrm{B'}\right) \tag{8.11}$$

- *In the case of* $i_d^{''}(t)$ *current* the direct voltage is calculated with the average value formula for the $T_p^{''}$ time period:

$$V_{d\alpha}^{''} = \frac{1}{\pi}\int_{2\pi+\alpha}^{3\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\cdot\left(Area\ \mathrm{A} + Area\ \mathrm{B''}\right) = f(\alpha, I_d) \tag{8.12}$$

As we can see in Fig.8.9 the average value $I_d^{''}$ corresponding to $i_d^{''}(t)$ current is greater than the average value $I_d^{'}$ which corresponds to $i_d^{'}(t)$ current ($I_d^{''} > I_d^{'}$) and:

$$\left.\begin{array}{l} \left|Area\ \mathrm{B''}\right| > \left|Area\ \mathrm{B'}\right| \\ Area\ \mathrm{B'} < 0,\ Area\ \mathrm{B''} < 0 \end{array}\right\} \Rightarrow V_{d\alpha}^{'} > V_{d\alpha}^{''} > 0 \tag{8.13}$$

The expression (8.13) shows the dependency of the average voltage provided by a rectifier operating in a discontinuous conduction mode on a random perturbation as the load current. Also, if a passive *L-R* load is used we have: |*Area* B'|< *Area* A. Thus, the output average voltage is always positive ($V_{d\alpha}^{'} > 0$), but lower than in the case of a purely resistive load. In the $U_{d\alpha}^{'}$ equation (8.9), *Area B*' dependent on the load current is missing for the resistive load, so the output DC voltage can be adjusted exclusively through the firing angle α.

The $i_d(t)$ current waveform can be determined by solving the voltage differential equation written for the thyristors' on-state time intervals:

$$R_d \cdot i_d(t) + L_d \frac{di_d(t)}{dt} = v_s(t) = \sqrt{2}\cdot V_s \sin\omega t \tag{8.14}$$

The equation (8.14) has two solutions, the first $i_{d1}(t) = K\cdot e^{-t/\tau}$ and the second: $i_{d2}(t) = \dfrac{\sqrt{2}\cdot V_s}{\sqrt{R_d^2 + (\omega L_d)^2}}\cdot\sin\left(\omega t - \varphi_d\right)$.

c) <u>Discontinuous conduction mode in case of a *R-L-E* load</u>

It is a common case in the adjustable electric drives with a DC motor supplied by a phased-controlled rectifier. Being a load that requires a filtered smooth current, the DC motor (M_{dc}) is in series with the filter inductance L_f (see Fig.8.10). Operating as a motor, the electrical DC machine rotates with n speed [rot/min] and generates an *emf* (electromotive force) voltage E (proportional with rotational speed $E=k_e \cdot \Phi_{ex} \cdot n$) *whose polarity is opposite to the supply voltage* $U_{d\alpha}$. Thus, the equivalent scheme of the load circuit, seen by the rectifier at the output, is of an *R-L-E* type, as shown in Fig.8.10 where: $L_d=L_f+L_a$ and $R_d=R_f+R_a$. L_a , R_a are the inductance and the resistance of DC motor armature, respectively.

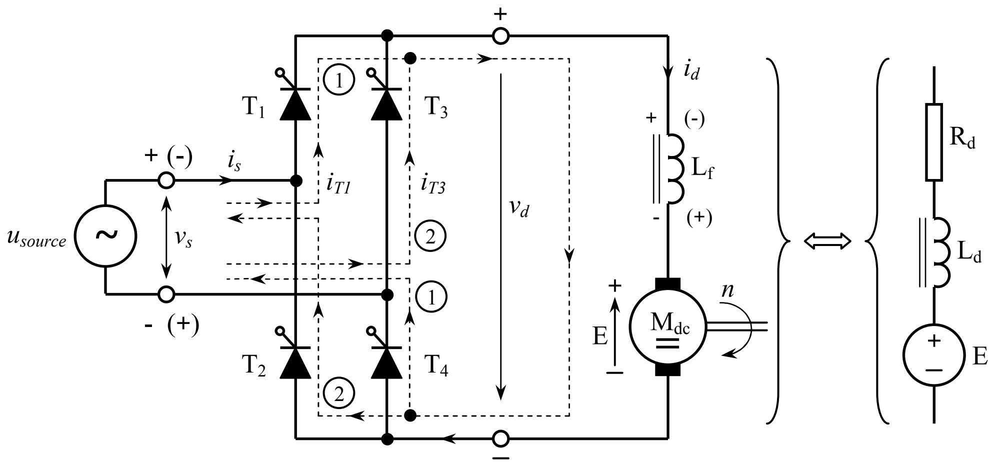

Fig.8.10 Phase-controlled rectifier with R-L-E load.

According to those presented above, the filter inductance value L_f is the result of a compromise between the size, mass and price of the coil, the ripple and the minimum value of the load current $i_d(t)$. If the load is a DC motor, the value of the average current I_d absorbed by the electrical machine is proportional with its shaft mechanical load. So, if this load decreases under a certain value, the load current decreases under a threshold value $I_d < I_d^*$ and the rectifier will operate in the discontinuous conduction mode. The waveforms corresponding to this mode in case of an *R-L-E* load are shown in Fig.8.11.

It should be noted that ***the active load favours the discontinuous conduction mode of the rectifier*** because the time moment whence the inductance begins to deliver its stored energy is outrun in the F points (see Fig.8.11) when the magnitude of the supply AC voltage decreases under the emf voltage E: $|u_s(t)| < E$. After these moments, the emf voltage tends to reverse bias the thyristors pair being in conduction and blocking the current flow. Due to the energy stored in the inductances field ($L_d = L_f + L_a$), with the current decreasing ($di_d / dt < 0$) an induced voltage appears:

$$v_L(t) = L_d \cdot \frac{di_d(t)}{dt} = |v_s(t)| - E - R_d \cdot i_d < 0 \tag{8.15}$$

whose polarity (+ on the bottom terminal of the L_d) and amplitude help the current to flow by compensating the difference between the magnitude of the AC voltage and the emf voltage plus the voltage drop on the R_d resistance.

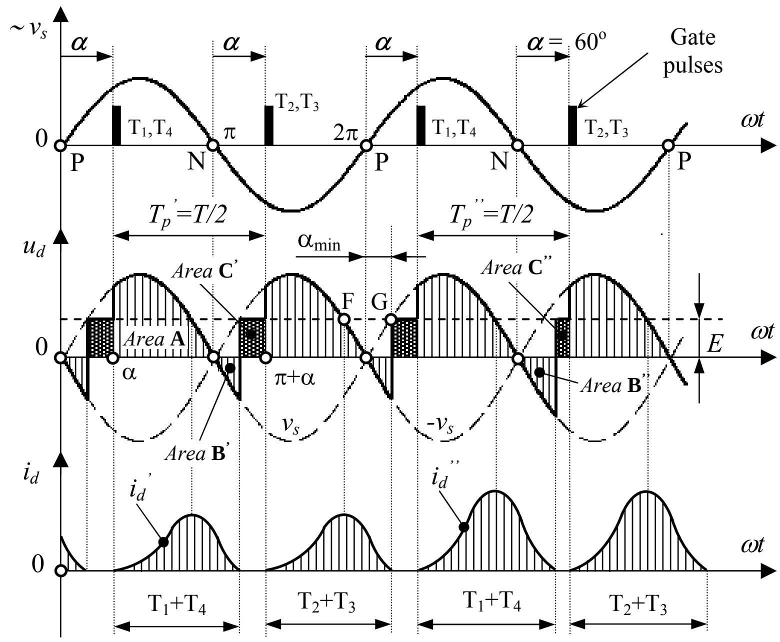

Fig. 8.11 Discontinuous conduction mode in the case of a single-phase rectifier with a *R-L-E* load (α=60° [el]).

When the motor mechanical load decreases below a certain value ($I_d < I_d^*$) or the filter inductance is undersized, the stored energy in the inductance is not sufficient to maintain the i_d current flow until the next thyristor pair is turned-on with a certain firing angle. Thus, discontinuous conduction mode appears with all associated negative issues:

- Abnormal function of the load (of the DC motor):
  - *noisy functioning* because of electromagnetic torque cancelling during the times intervals where the current is zero.
  - *increasing losses in the DC motor* (iron losses and Joule losses) due to the AC component of the current over the DC component. Due to the increasing losses, the electrical machine can overheat and that's why we choose DC motors specially designed to be supplied by power electronic converters, having a magnetic circuit formed by plates and a slightly increased power.

- The rectifier becomes uncontrollable from the output DC voltage point of view, because it depends, besides the firing angle, on the load current I_d. The phenomenon has been explained in the previous section in the case of the discontinuous current mode for an *R-L* load. If we comparatively look at Fig.8.9 and Fig.8.11 we see that in discontinuous current mode, for a *R-L-E* load, the *v_d output voltage waveform contains sections of electromotive voltage E during the time intervals in which the i_d current is zero.* It is an indication that the rectifier is in the discontinuous current mode if only the output voltage waveform is displayed with the help of an oscilloscope.

The DC voltage dependence on the load current results from Fig.8.11 where the waveforms for two different currents $i_d^{'}(t)$ and $i_d^{''}(t)$ are shown:

- *In the case of the $i_d^{'}(t)$ current,* the direct voltage is calculated with the average value formula for the $T_p^{'}$ time interval (pulse time period):

$$V_{d\alpha}^{'} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\cdot\left(Area\ \mathrm{A} + Area\ \mathrm{B}^{'} + Area\ \mathrm{C}^{'}\right) \quad (8.16)$$

- *In the case of the $i_d^{''}(t)$ current* the direct voltage is calculated with the average value formula for the $T_p^{''}$ time period:

$$V_{d\alpha}^{''} = \frac{1}{\pi}\int_{2\pi+\alpha}^{3\pi+\alpha} v_d(\omega t)\cdot d(\omega t) = \frac{1}{\pi}\cdot\left(Aria\ \mathrm{A} + Aria\ \mathrm{B}^{''} + Aria\ \mathrm{C}^{''}\right) \quad (8.17)$$

Taking into consideration that the average values $I_d^{''} > I_d^{'}$ results:

$$\left.\begin{array}{l} Area\ \mathrm{B}^{''} < Area\ \mathrm{B}^{'} < 0 \\ 0 < Area\ \mathrm{C}^{''} < Area\ \mathrm{C}^{'} \end{array}\right\} \Rightarrow 0 < V_{d\alpha}^{''} < V_{d\alpha}^{'} \quad (8.18)$$

- Limiting the firing angle to a minimum value to obtain the possibility of the thyristors' turn-on:

$$\alpha > \alpha_{\min} \quad (8.19)$$

As the firing angle decreases, the average value of the output voltage increases leading to the increase of the motor speed *n* and also of emf voltage *E*. Thus, the firing angle value comes rapidly near to the points G that are found at the intersections between *E* and $|v_s(t)|$ waveforms (see Fig.8.11). If the firing angle becomes $\alpha < \alpha_{\min} \Rightarrow |v_s(t)| < E$ the emf voltage (voltage of the active load) reverse bias the thyristors which follow to be turned on. Because of the short gate pulses, these thyristors remain in off state more time intervals T_p in which there is no DC voltage at the rectifier's output. During these off intervals the motor is not supplied and the speed decrease includes the emf voltage E. At some moment: $E < \left|v_s(\omega t)\right|_{\omega t=\alpha}$ the thyristors can also be turned on, thus supplying again the DC motor. The motor increases its speed, respectively the emf voltage E reaching once more the situation of reverse bias of the thyristors. The phenomenon begins to repeat and an ***oscillation of the rectifier-motor system*** appears. This can be avoided if a continuous firing of the thyristors gate is made (wide gate current pulses) after the moment $\omega t = \alpha$. Thus, it is obtained a diode rectifier behaviour, diodes which are turned on as soon as the input AC voltage meets the requirement: $\left|v_s(t)\right| > E$.

Besides the negative consequences mentioned above which appear when the rectifier operates in a steady state discontinuous conduction mode, it can also be highlighted an oscillation of the rectifier-motor system when the rectifier passes from continuous conduction to discontinuous conduction mode.

According to the previous, for a certain filtering induction L_f and for a certain firing angle, if the motor mechanical load drops below a threshold value ($I_d < I_d^*$), the rectifier goes from the continuous conduction to the discontinuous conduction mode. This transition occurs with a significant increase of the output DC voltage:

$$V_{d\alpha} \rightarrow V'_{d\alpha} = V_{d\alpha} + \Delta V_d \quad \text{where } \Delta V_d = f(\alpha, I_d) \tag{8.20}$$

because a part from the negative B area corresponding to the continuous conduction mode (see Fig.3.11) is turned into the positive area C'. Due to sudden increase of the output DC voltage, the average value of the motor current will also increase faster depending on the electromagnetic time constant because the speed (E) increases slower, following the electromechanical time constant.

$$V'_{d\alpha} = V_{d\alpha} + \Delta V_d = E + R_d\left(I_d + \Delta I_d\right) \tag{8.21}$$

The increasing with ΔI_d of the I_d current causes the acceleration of the motor (the electromagnetic torque increases) and therefore the rectifier switches again in the continuous conduction mode because:

$$I_d + \Delta I_d > I_d^* \tag{8.22}$$

By switching the rectifier into continuous conduction mode, its output DC voltage decreases suddenly to the value $V_{d\alpha}$ which causes the decrease of the average current I_d and a motor deceleration. After a while comes back the situation in which $I_d < I_d^*$. The rectifier passes again in the discontinuous conduction mode, the processes being rerun as shown above, in an oscillation that is dangerous for both the motor and the rectifier.

We can observe that the rectifier's discontinuous conduction mode for an active load is more harmful than for an R-L passive load, especially if the active load is a DC motor (R-L-E). Because of this, in such applications it is calculated and chosen carefully the filter inductance value to avoid the discontinuous conduction mode even for the smallest mechanical loads of the electrical motor. In addition, if the rectifier and the motor is part of a regulated electric drive, the discontinuous conduction mode appearance at a certain time can lead to system instability because the controllers (regulators) have been tuned considering the normal operating mode of the rectifier (continuous conduction mode).

In conclusion, it can be said that the discontinuous current mode is an abnormal one in the operating mode of the phase-controlled rectifier with *R-L* or *R-L-E* load. Its study is required to know how it can be avoided or if it appears which measures should be taken to reduce the negative influences of the consequences above mentioned.

6. Laboratory application

In order to experimentally study the aspects related to the operating mode of a single-phase controlled rectifier, it will be performed the laboratory setup with the block diagram shown in Fig.8.12 and image in Fig.13. For the rectifier load two variants are used: a passive load *R* and *R-L*, respectively an active *R-L-E* load. For this, in the laboratory are available the following circuit components:

- a line-frequency transformer (TR) to supply the phase-controlled rectifier with a low voltage ($24V_{ac}$);
- a single-phase full-bridge rectifier (B2) with thyristors made as a power integrated module (PIM). The module is placed on a radiator on which there are placed also the connecting power terminals and the control terminals (banana socket connectors;
- a gate trigger circuit (GTC) for all the thyristors included in the full-bridge structure. This phase control circuit is described in Lab no.3 and achieved with the help of the specialized integrated circuit UAA145;
- a DC motor (*M*) with permanent magnets mechanical coupled on the same shaft with an identical electrical machine that operates in generator mode (*G*). The role of the generator is to load mechanically the motor and therefore to obtain a variation of the DC output current provided by the phase-controlled rectifier to study the discontinuous conduction mode of the rectifier for different load currents;
- an autotransformer (*ATR*) connected between 0 and the cursor terminal. Thus, the autotransformer is in position of the filter inductance L_f whose value can be continuously adjusted;
- a rheostat (connected between 0 terminal and the cursor) in the position of the variable resistance R_d or in the position of the brake resistance R_{br} connected as load for the DC generator ;
- a shunt sensor with its help can be displayed the waveform of the i_d current;

All above mentioned circuit elements can be interconnected through the cables having the banana plugs, as is shown in the image from Fig.8.13. It will be used a voltmeter to measure the average value of the output voltage and an oscilloscope with two spots to display the waveforms of v_d and i_d.

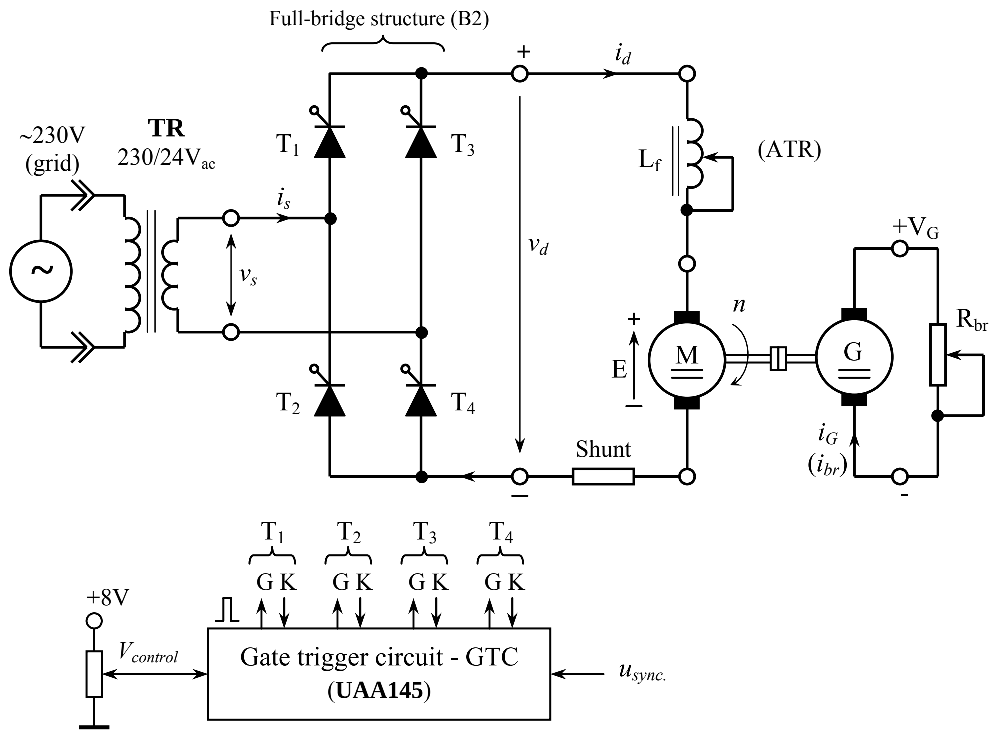

Fig. 8.12 Block diagram of the laboratory setup – single-phase bridge rectifier (B2) with an *R-L-E* active load.

The *R-L* load can be obtained by connecting a rheostat in the place of the DC motor shown in Fig.8.12. Purely resistive load is obtained by bringing the cursor of ATR in 0 position. Because the gate trigger circuit and the power structure are supplied from the power grid through different plug cables, when they are connected to the grid we must be careful to realise the correct phasing of the gate trigger pulses. If the setup doesn't work, one of the plugs will be reversed in the socket.

7. Objectives and procedures

1. It will be studied the theoretical operation aspects of the single-phase bridge rectifier with a current filter at the output; waveforms, the average output voltage equation, adjusting characteristic etc.

2. It will be studied the theoretical aspects related to the discontinuous conduction mode of a bridge phase-controlled rectifier for different types of loads and the consequences that appear due to this mode;
3. It will be performed the laboratory setup with block diagram from Fig.8.12, it will be started the operation of the rectifier in continuous conduction mode (great L_f) and it will be visualised the waveforms v_d and i_d for different firing angles in rectifier mode (α<90^o);
4. It will be measured the average value (DC voltage $V_{d\alpha}$) of the output voltage v_d for different firing angles using a voltmeter and it will be observed the motor speed variation with the changed value of the DC voltage;
5. It will be performed the power structures from Fig.8.5, Fig.8.8 and Fig.8.10 and it will be displayed the waveforms corresponding to each structure (each load type) in discontinuous conduction mode (see Fig.8.6, Fig.8.9 and Fig.8.11);
6. It will be observed the motor behaviour in discontinuous current mode;
7. It will be highlighted the increase of average voltage at the rectifier's output when the discontinuous conduction mode occurs and also the voltage variation with the output DC current (I_d) when the mechanical load of motor is changed with the help of the DC generator (the generator is loaded by decreasing the brake resistance R_{br} connected to its terminals);
8. It will be highlighted the oscillation of the rectifier-motor system when the firing angle is progressively decreased in discontinuous conduction mode;

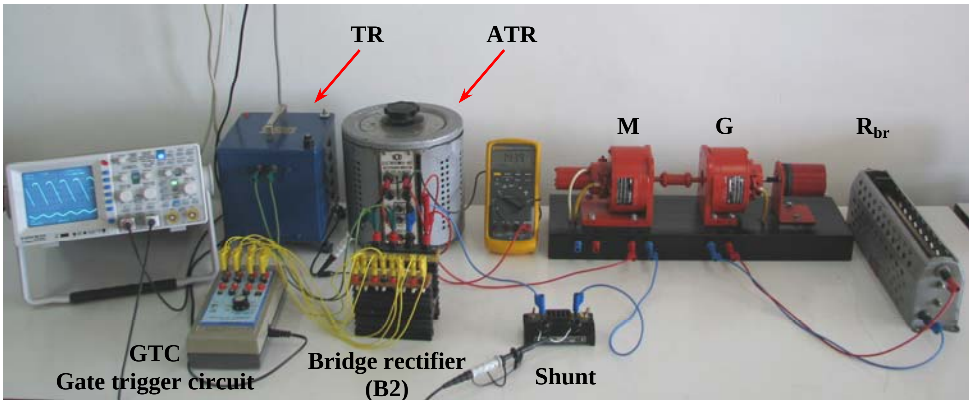

Fig. 8.13 Image of the laboratory application.

References:

[1] Mohan N., Undeland T., Robbins W., *Power Electronics: Converters, Applications and Design*, Third Edition, Published by John Willey &Sons Inc., USA, 2003.

[2] Albu M., *Electronică de putere* - vol I: *Noțiuni introductive, dispozitive, conversia statică alternativ-continuu a energiei electrice*, Casa de Editură "Venus" Iaşi, 2007.

[3] Diaconescu M.P., Graur I.: *Convertoare statice – baze teoretice, elemente de proiectare, aplicații*, Ed. „Gh. Asachi”, Iași, 1996.
[4] Ionescu Fl., Floricău D., Nițu S., Six J.P, Delarue Ph., Boguş C.: *Electronică de putere - convertoare statice*, Ed. Tehnică, Bucureşti, 1998.
[5] Kelemen A., Imecs M., *Electronică de putere*, Ed. Didactică şi Pedagogică, Bucureşti, 1983.